

1-2-3 Method – Integration by Parts

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In Figure 1.1., the 1-2-3 Method for simplified Integration by Parts is presented as an improved shortcut for actuarial students to facilitate passing Exam P. Only 3 calculus steps are required (Figure 1.1.).

Figure 1.1. 1-2-3 Method – Integration by Parts

$$f(x) * \int g(x) - \int \left[\frac{d}{dx} f(x) * \int g(x) \right]$$

The formula simplifies traditional integration by parts, a tedious and time consuming calculus process, reducing errors by half, and speeding calculation by up to 200-300%. Therefore, calculation of the antiderivative of $g(x)$ occurs only once, substituted back into the first $g(x)$. Although loosely based on an existing user-unfriendly calculus method, to the best of the author's knowledge, this is the first incidence of the more precise expression, in the actuarial sciences. The key advantage of the 1-2-3 Method is that only 2 of the 4 functions require calculus calculations, in 3 simple steps (expression 1.1.):

$$f(x) * \int g(x) - \int \left[\frac{d}{dx} f(x) * \int g(x) \right] \quad 1.1.$$

Example:

Previous SOA Exam P tests included several similar problems:

Find $\int x e^{-2x} dx = F(x) = ?$

Define $f(x) = x$ and $g(x) = e^{-2x}$.

Then it follows: $\frac{d}{dx}f(x) = 1$, $\int g(x) = -.5e^{-2x}$.

Solution:

Calculus Steps 1 & 2:

Take the derivative of the second $f(x)$ and antiderivative of the second $g(x)$ (expression 1.2.):

$$f(x) * \int g(x) - \int \left[\frac{d}{dx} f(x) * \int g(x) \right] \quad 1.2$$

$$f(x) * \int g(x) - \int [1 * -.5e^{-2x}]$$

Next, substitute the $-.5e^{-2x}$ result into the first $g(x)$:

$$f(x) * -.5e^{-2x} - \int [1 * -.5e^{-2x}]$$

, and $f(x) = x$ remains unchanged:

$$x * -.5e^{-2x} - \int [1 * -.5e^{-2x}]$$

Calculus Step 3:

Find the **antiderivative** of the product:

$$x * -.5e^{-2x} - \int [1 * -.5e^{-2x}]$$

$$x * -.5e^{-2x} - .25e^{-2x}$$

Simplify:

$$-.5xe^{-2x} - .25e^{-2x} =$$

$$\underline{-.25e^{-2x}(2x + 1)}$$

Compressed Steps:

$$f(x) * \int g(x) - \int \left[\frac{d}{dx} f(x) * \int g(x) \right] =$$

$$x * -.5e^{-2x} - \int [1 * -.5e^{-2x}] =$$

$$-.5xe^{-2x} - \int[-.5e^{-2x}] =$$

$$-.5xe^{-2x} - .25e^{-2x} =$$

$$\underline{-.25e^{-2x}(2x + 1)}$$